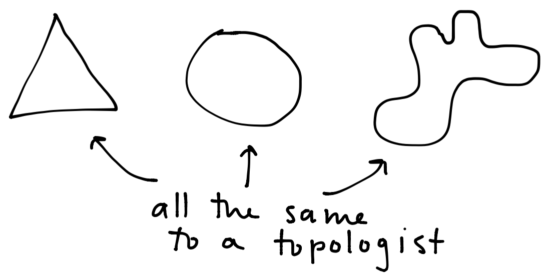
What is topology?

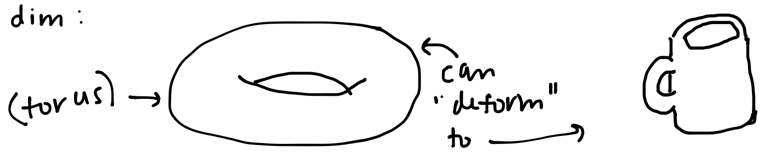
Geometry but more "rubbery".

If X is a set, a topology on X is necessary to study properties that are invariant under continuous deformation.

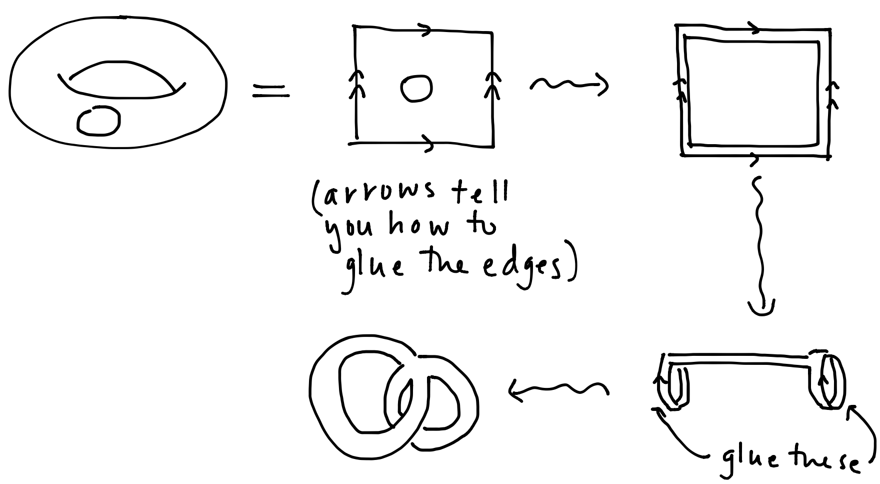
Ex: 1 dim:



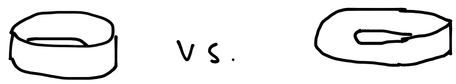
2 dim:

1.) 

2.) What if we cut a hole in a torus and tried to flatten it?



3.) Is an ordinary band of paper different topologically from a Möbius band?



4.) Is two linked bands different topologically from two unlinked bands?

Topological spaces

- Idea: the property we want to maintain in a topological space is "nearness". Can bend, stretch objects, but not tear them. This is described using open sets.

- In most areas of math, we study objects whose underlying set is a topological space equipped w/ additional structur (not always metric spaces).

- Topological spaces are the broadest set of objects where we can define continuous functions.

Ex: In calculus, extreme value thm says:

If $f: [a,b] \to \mathbb{R}$ is continuous, then $\exists\ x \in [a,b]$ where f achieves its max (or min).

However, this is true for any continuous $f: X \to \mathbb{R}$ where X is a compact topological space.

Invariants

How do we tell topological spaces apart?

One way: assign invariants to help us to distinguish them — e.g. dimension, orientability, fundamental group, connectedness

Algebraic topology gives us some tools w/ which to approach this.

This class:

Point-set topology $\} \leq 2/3$
Algebraic topology $\} \geq 1/3$